MATLAB Function Reference

Provide feedback about this page

pdepe

Solve initial-boundary value problems for parabolic-elliptic PDEs in 1-D

Syntax

```
sol = pdepe(m,pdefun,icfun,bcfun,xmesh,tspan)
sol = pdepe(m,pdefun,icfun,bcfun,xmesh,tspan,options)
```

Arguments

m	A parameter corresponding to the symmetry of the problem. m can be slab = 0, cylindrical = 1, or spherical = 2.
pdefun	A handle to a function that defines the components of the PDE.
icfun	A handle to a function that defines the initial conditions.
bcfun	A handle to a function that defines the boundary conditions.
xmesh	A vector [x0, x1, ..., xn] specifying the points at which a numerical solution is requested for every value in tspan. The elements of xmesh must satisfy x0 < x1 < ... < xn. The length of xmesh must be >= 3.
tspan	A vector [t0, t1, ..., tf] specifying the points at which a solution is requested for every value in xmesh. The elements of tspan must satisfy t0 < t1 <... < tf. The length of tspan must be >= 3.
options	Some options of the underlying ODE solver are available in pdepe: RelTol, AbsTol, NormControl, InitialStep, and MaxStep. In most cases, default values for these options provide satisfactory solutions. See odeset for details.

Description

`sol = pdepe(m,pdefun,icfun,bcfun,xmesh,tspan)` solves initial-boundary value problems for systems of parabolic and elliptic PDEs in the one space variable x and time t. `pdefun`, `icfun`, and `bcfun` are function handles. See Function Handles in the MATLAB Programming documentation for more information. The ordinary differential equations (ODEs) resulting from discretization in space are integrated to obtain approximate solutions at times specified in `tspan`. The `pdepe` function returns values of the solution on a mesh provided in `xmesh`.

Parameterizing Functions Called by Function Functions, in the MATLAB Mathematics documentation, explains how to provide additional parameters to the functions `pdefun`, `icfun`, or `bcfun`, if necessary.

`pdepe` solves PDEs of the form:

$$c\left(x,t,u,\frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial t} = x^{-m}\frac{\partial}{\partial x}\left(x^{m} f\left(x,t,u,\frac{\partial u}{\partial x}\right)\right) + s\left(x,t,u,\frac{\partial u}{\partial x}\right) \quad \textbf{(2-2)}$$

The PDEs hold for $t_0 \le t \le t_f$ and $a \le x \le b$. The interval $[a, b]$ must be finite. m can be 0, 1, or 2, corresponding to slab, cylindrical, or spherical symmetry, respectively. If $m > 0$, then a must be >= 0.

In Equation 2-2, $f(x, t, u, \partial u/\partial x)$ is a flux term and $s(x, t, u, \partial u/\partial x)$ is a source term. The coupling of the partial derivatives with respect to time is restricted to multiplication by a diagonal matrix $c(x, t, u, \partial u/\partial x)$. The diagonal elements of this matrix are either identically zero or positive. An element that is identically zero corresponds to an elliptic equation and otherwise to a parabolic equation. There must be at least one parabolic equation. An element of c that corresponds to a parabolic equation can vanish at isolated values of x if those values of x are mesh points. Discontinuities in c and/or s due to material interfaces are permitted provided that a mesh point is placed at each interface.

For $t = t_0$ and all x, the solution components satisfy initial conditions of the form

$$u(x, t_0) = u_0(x) \quad \textbf{(2-3)}$$

For all t and either $x = a$ or $x = b$, the solution components satisfy a boundary condition of the form

(2-4)

$$p(x,t,u) + q(x,t)\,f\left(x,t,u,\frac{\partial u}{\partial x}\right) = 0$$

Elements of q are either identically zero or never zero. Note that the boundary conditions are expressed in terms of the flux f rather than $\partial u/\partial x$. Also, of the two coefficients, only p can depend on u.

In the call `sol = pdepe(m,pdefun,icfun,bcfun,xmesh,tspan)`:

- `m` corresponds to m.
- `xmesh(1)` and `xmesh(end)` correspond to a and b.
- `tspan(1)` and `tspan(end)` correspond to t_0 and t_f.
- `pdefun` computes the terms c, f, and s (Equation 2-2). It has the form

  ```
  [c,f,s] = pdefun(x,t,u,dudx)
  ```

  The input arguments are scalars `x` and `t` and vectors `u` and `dudx` that approximate the solution u and its partial derivative with respect to x, respectively. `c`, `f`, and `s` are column vectors. `c` stores the diagonal elements of the matrix c (Equation 2-2).
- `icfun` evaluates the initial conditions. It has the form

  ```
  u = icfun(x)
  ```

  When called with an argument `x`, `icfun` evaluates and returns the initial values of the solution components at `x` in the column vector `u`.
- `bcfun` evaluates the terms p and q of the boundary conditions (Equation 2-4). It has the form

  ```
  [pl,ql,pr,qr] = bcfun(xl,ul,xr,ur,t)
  ```

  `ul` is the approximate solution at the left boundary `xl` = a and `ur` is the approximate solution at the right boundary `xr` = b. `pl` and `ql` are column vectors corresponding to p and q evaluated at `xl`, similarly `pr` and `qr` correspond to `xr`. When $m > 0$ and $a = 0$, boundedness of the solution near $x = 0$ requires that the flux f vanish at $a = 0$. `pdepe` imposes this boundary condition automatically and it ignores values returned in `pl` and `ql`.

`pdepe` returns the solution as a multidimensional array `sol`. u_i = `ui` =`sol(:,:,i)` is an approximation to the `i`th component of the solution vector u. The element `ui(j,k)` = `sol(j,k,i)` approximates u_i at (t, x) = (`tspan(j)`,`xmesh(k)`).

`ui = sol(j,:,i)` approximates component `i` of the solution at time `tspan(j)` and mesh points `xmesh(:)`. Use `pdeval` to compute the approximation and its partial derivative $\partial u_i/\partial x$ at points not included in `xmesh`. See `pdeval` for details.

`sol = pdepe(m,pdefun,icfun,bcfun,xmesh,tspan,options)` solves as above with default integration parameters replaced by values in `options`, an argument created with the `odeset` function. Only some of the options of the underlying ODE solver are available in `pdepe`: `RelTol`, `AbsTol`, `NormControl`, `InitialStep`, and `MaxStep`. The defaults obtained by leaving off the input argument `options` will generally be satisfactory. See `odeset` for details.

Remarks

- The arrays `xmesh` and `tspan` play different roles in `pdepe`.

  `tspan` – The `pdepe` function performs the time integration with an ODE solver that selects both the time step and formula dynamically. The elements of `tspan` merely specify where you want answers and the cost depends weakly on the length of `tspan`.

  `xmesh` – Second order approximations to the solution are made on the mesh specified in `xmesh`. Generally, it is best to use closely spaced mesh points where the solution changes rapidly. `pdepe` does *not* select the mesh in x automatically. You must provide an appropriate fixed mesh in `xmesh`. The cost depends strongly on the length of `xmesh`. When $m > 0$, it is not necessary to use a fine mesh near $x = 0$ to account for the coordinate singularity.
- The time integration is done with `ode15s`. `pdepe` exploits the capabilities of `ode15s` for solving the differential-algebraic equations that arise when Equation 2-2 contains elliptic equations, and for handling Jacobians with a specified sparsity pattern.
- After discretization, elliptic equations give rise to algebraic equations. If the elements of the initial conditions vector that correspond to elliptic equations are not "consistent" with the discretization, `pdepe` tries to adjust them before beginning the time integration. For this reason, the solution returned for the initial time may have a discretization error comparable to that at any other time. If the mesh is sufficiently fine, `pdepe` can find consistent

initial conditions close to the given ones. If `pdepe` displays a message that it has difficulty finding consistent initial conditions, try refining the mesh.

No adjustment is necessary for elements of the initial conditions vector that correspond to parabolic equations.

Examples

Example 1. This example illustrates the straightforward formulation, computation, and plotting of the solution of a single PDE.

$$\pi^2 \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$$

This equation holds on an interval $0 \le x \le 1$ for times $t \ge 0$.

The PDE satisfies the initial condition

$$u(x, 0) = \sin \pi x$$

and boundary conditions

$$u(0, t) \equiv 0$$

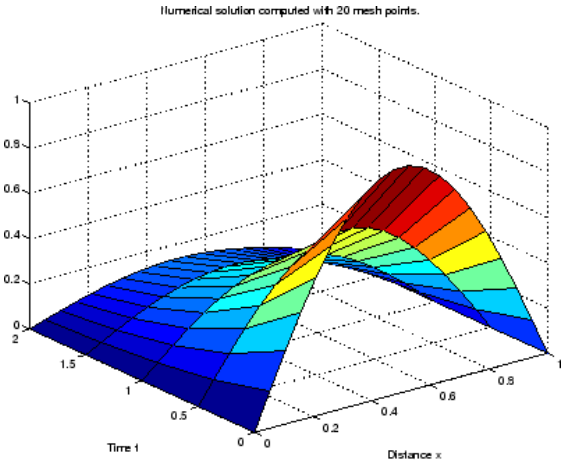
$$\pi e^{-t} + \frac{\partial u}{\partial x}(1, t) = 0$$

It is convenient to use subfunctions to place all the functions required by `pdepe` in a single M-file.

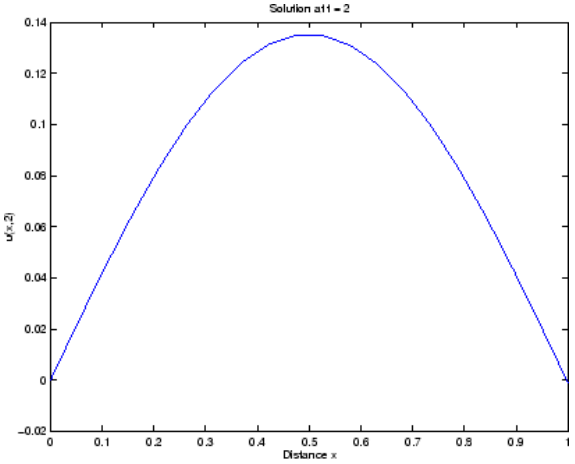
```
function pdex1

m = 0;
x = linspace(0,1,20);
t = linspace(0,2,5);

sol = pdepe(m,@pdex1pde,@pdex1ic,@pdex1bc,x,t);
% Extract the first solution component as u.
u = sol(:,:,1);

% A surface plot is often a good way to study a solution.
surf(x,t,u)
title('Numerical solution computed with 20 mesh points.')
xlabel('Distance x')
ylabel('Time t')

% A solution profile can also be illuminating.
figure
plot(x,u(end,:))
title('Solution at t = 2')
xlabel('Distance x')
ylabel('u(x,2)')
% --------------------------------------------------------------
function [c,f,s] = pdex1pde(x,t,u,DuDx)
c = pi^2;
f = DuDx;
s = 0;
% --------------------------------------------------------------
function u0 = pdex1ic(x)
u0 = sin(pi*x);
% --------------------------------------------------------------
function [pl,ql,pr,qr] = pdex1bc(xl,ul,xr,ur,t)
pl = ul;
ql = 0;
pr = pi * exp(-t);
qr = 1;
```

In this example, the PDE, initial condition, and boundary conditions are coded in subfunctions `pdex1pde`, `pdex1ic`, and `pdex1bc`.

The surface plot shows the behavior of the solution.



The following plot shows the solution profile at the final value of `t` (i.e., `t =2`).



Example 2. This example illustrates the solution of a system of PDEs. The problem has boundary layers at both ends of the interval. The solution changes rapidly for small t.

The PDEs are

$$\frac{\partial u_1}{\partial t} = 0.024 \frac{\partial^2 u_1}{\partial x^2} - F(u_1 - u_2)$$

$$\frac{\partial u_2}{\partial t} = 0.170 \frac{\partial^2 u_2}{\partial x^2} + F(u_1 - u_2)$$

where $F(y) = \exp(5.73y) - \exp(-11.46y)$.

This equation holds on an interval $0 \le x \le 1$ for times $t \ge 0$.

The PDE satisfies the initial conditions

$$u_1(x, 0) \equiv 1$$

$$u_2(x, 0) \equiv 0$$

and boundary conditions

$$\frac{\partial u_1}{\partial x}(0, t) \equiv 0$$

$$u_2(0, t) \equiv 0$$

$$u_1(1, t) \equiv 1$$

$$\frac{\partial u_2}{\partial x}(1, t) \equiv 0$$

In the form expected by `pdepe`, the equations are

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} .* \frac{\partial}{\partial t}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \frac{\partial}{\partial x}\begin{bmatrix} 0.024(\partial u_1/\partial x) \\ 0.170(\partial u_2/\partial x) \end{bmatrix} + \begin{bmatrix} -F(u_1 - u_2) \\ F(u_1 - u_2) \end{bmatrix}$$

The boundary conditions on the partial derivatives of u have to be written in terms of the flux. In the form expected by `pdepe`, the left boundary condition is

$$\begin{bmatrix} 0 \\ u_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} .* \begin{bmatrix} 0.024(\partial u_1/\partial x) \\ 0.170(\partial u_2/\partial x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and the right boundary condition is

$$\begin{bmatrix} u_1 - 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} .* \begin{bmatrix} 0.024(\partial u_1/\partial x) \\ 0.170(\partial u_2/\partial x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The solution changes rapidly for small t. The program selects the step size in time to resolve this sharp change, but to see this behavior in the plots, the example must select the output times accordingly. There are boundary layers in the solution at both ends of [0,1], so the example places mesh points near `0` and `1` to resolve these sharp changes. Often some experimentation is needed to select a mesh that reveals the behavior of the solution.

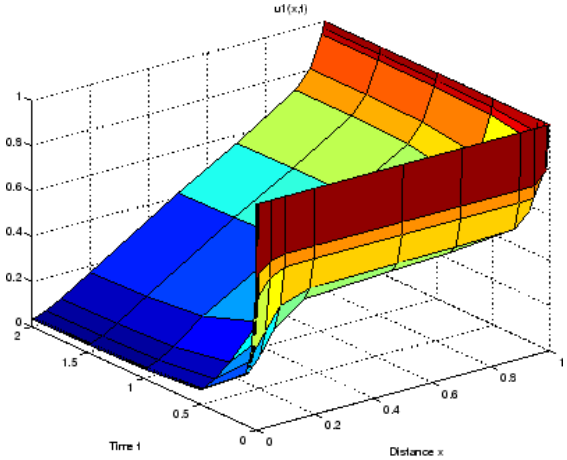
```
function pdex4
m = 0;
x = [0 0.005 0.01 0.05 0.1 0.2 0.5 0.7 0.9 0.95 0.99 0.995 1];
t = [0 0.005 0.01 0.05 0.1 0.5 1 1.5 2]; %uneven for better figures

sol = pdepe(m,@pdex4pde,@pdex4ic,@pdex4bc,x,t);
u1 = sol(:,:,1);
u2 = sol(:,:,2);

figure
surf(x,t,u1)
title('u1(x,t)')
xlabel('Distance x')
ylabel('Time t')

figure
surf(x,t,u2)
title('u2(x,t)')
xlabel('Distance x')
ylabel('Time t')
% --------------------------------------------------------------
function [c,f,s] = pdex4pde(x,t,u,DuDx)
c = [1; 1];
f = [0.024; 0.17] .* DuDx; %array multiplier. e.g. [3 4].*2=[6 8]
```

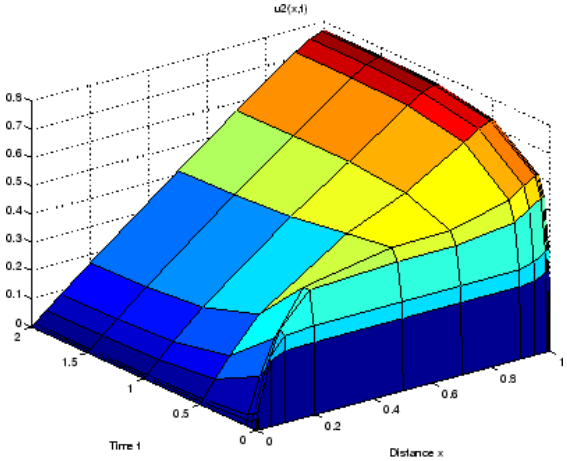


```
y = u(1) - u(2);
F = exp(5.73*y)-exp(-11.47*y);
s = [-F; F];
% --------------------------------------------------------------
function u0 = pdex4ic(x);
u0 = [1; 0];
% --------------------------------------------------------------
function [pl,ql,pr,qr] = pdex4bc(xl,ul,xr,ur,t)
pl = [0; ul(2)];
ql = [1; 0];
pr = [ur(1)-1; 0];
qr = [0; 1];
```

In this example, the PDEs, initial conditions, and boundary conditions are coded in subfunctions `pdex4pde`, `pdex4ic`, and `pdex4bc`.

The surface plots show the behavior of the solution components.





See Also

`function_handle (@)`, `pdeval`, `ode15s`, `odeset`, `odeget`

References

[1] Skeel, R. D. and M. Berzins, "A Method for the Spatial Discretization of Parabolic Equations in One Space Variable," *SIAM Journal on Scientific and Statistical Computing*, Vol. 11, 1990, pp.1-32.

Provide feedback about this page

pcolor pdeval