ChE 101: Working with Matrices and Arrays
Lesson 1: Matrix and Vector Operations
(H-3)

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Learning Objectives

- Lesson 1:
  - Define matrices
  - Distinguish vectors and matrices
  - Identify square matrices
  - Perform basic matrix operations
    - Sum/subtraction
    - Multiply matrices

Review of Matrices
- **Definition**
- Types of matrices
- Matrix operations

Matrix Definition

- Array of numbers that can be represented in the following form:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1m} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nm} \end{bmatrix}$$

row

column

- Where “n” represents the row number
- Where “m” represents the column number
- Matrix A has dimensions (n x m)

Exercise 1

- Given the following matrices, identify the dimensions of each matrix, what is the value of the following elements a_{12}, a_{21}, a_{32},a_{22}

$$A = \begin{bmatrix} 2 & 0.5 & 3 \\ 1.5 & 2.2 & -1 \\ 0 & 10 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 4 & 7 \\ 2 & 15 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

Review of Matrices
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Types of Matrices

- There are different types of important matrices in linear algebra
- However, for the purpose of this course we will focus on:
  - Vectors
  - Square matrices

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Vectors

- Vectors are type of matrices that have either one row or one column
- Matrices with one column (m=1) are called column vectors, e.g.,

$$\{A\} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

- Matrices with one row (n=1) are called row vectors, e.g.,

$$\{B\} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Square matrices

Review of Matrices
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- If m=n, we have a square matrix, eg:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & 2 & 0.5 \\ 10 & 15 & 2 & 3.5 \\ 4 & 1 & 2 & 0.4 \end{bmatrix}$$

Matrix Operations

Review of Matrices
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- Addition:
  - Matrices can be added only if they have the same dimensions
  - To add matrices each element must be added
  - Steps for adding matrices:
    - Identify the dimensions of the matrices
    - If the dimensions are not the same the matrices can't be added
    - If the dimensions are the same the matrices can be added by adding each of the individual elements

Example Adding Matrices

- Given A and B, calculate C= A+B

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 2.5 \\ -3 & -2 \end{bmatrix}$$

- Procedure:
  - Dimensions of A: 2x2, Dimensions of B: 2x2
  - Because both matrices have the same dimensions they can be added
  - Elements of C matrix:
    - $c_{11}=a_{11}+b_{11}$ = 1+0 =1
    - $c_{12}=a_{12}+b_{12}$ = 2+2.5 =4.5
    - $c_{21}=a_{21}+b_{21}$ = 3-3 =0
    - $c_{22}=a_{22}+b_{22}$ = 4-2 =2

$$C = \begin{bmatrix} 1 & 4.5 \\ 0 & 2 \end{bmatrix}$$

Exercise 2

- Given the matrices, A, B, and C obtain matrix D = A + B and E=B+C

$$A=\begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}\quad B=\begin{bmatrix}2 & 3\\4 & 1\end{bmatrix}\quad \mathrm{C}=\begin{bmatrix}2 & 2 & 1\\0 & 4 & -1\\2 & 5 & 0.5\end{bmatrix}$$

Review of Matrices
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Matrix Operations

- Subtraction:
  - Matrices can be subtracted only if they have the same dimensions
  - To subtract matrices each element must be subtracted
  - Steps for subtracting matrices:
    - Identify the dimensions of the matrices
    - If the dimensions are not the same the matrices can't be subtracted
    - If the dimensions are the same the matrices can be subtracted by subtracting each of the individual elements

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Example Subtracting Matrices

- Given A and B, calculate C= A-B

$$A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\quad B=\begin{bmatrix}0 & 2.5\\-3 & -2\end{bmatrix}$$

- Procedure:
  - Dimensions of A: 2x2, Dimensions of B: 2x2
  - Because both matrices have the same dimensions they can be subtracted
  - Elements of C matrix:
    - $c_{11}=a_{11}-b_{11}$ = 1-0 =1
    - $c_{12}=a_{12}-b_{12}$ = 2-2.5 =-0.5
    - $c_{21}=a_{21}-b_{21}$ = 3-(-3) =6
    - $c_{22}=a_{22}-b_{22}$ = 4-(-2) =6

$$C=\begin{bmatrix}1 & -0.5\\6 & 6\end{bmatrix}$$

Exercise 3

- Given the matrices, A and B, obtain matrix C = A - B

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

Matrix Operations

Review of Matrices
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- Multiplication of a matrix by a scalar:
  - If “g” is a scalar, a matrix “A” can be multiply by “g”
  - To do this, we need to multiply each element of matrix “A” by the scalar “g”
  - See procedure below:

$$C = g \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

scalar matrix

$$C = \begin{bmatrix} g * a_{11} & g * a_{12} \\ g * a_{21} & g * a_{22} \end{bmatrix}$$

Exercise 4

- Given the matrices A, B, C, D
- Perform the following calculations
  - F=A-B+0.5D
  - G=2(A+B+C)-D

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix}$$

Review of Matrices
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Matrix Operations

- Product of two matrices
  - The number of columns in the first matrix (m_1), must be equal to the number of rows in the second matrix (n_2). If the conditions given above are not true, the two matrices can't be multiply
  - The new matrix will have dimensions: n_1m_2
  - Multiply each element of the row of the first matrix, by each element of the column of the second matrix and add them. This operation will generate the new elements of the product matrix

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Review of Matrices
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Example Product

- Given Matrices A and B, calculate C=A*B

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

Solution:

Dimensions of A: 2x2 $\leftarrow$ $m_1=2$

Dimensions of B: 3x2 $\leftarrow$ $n_2=3$

Because m_1 is different to n_2 the product can't be done

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Example Product

- Given Matrices A and B, calculate C=A*B

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Solution:

Dimensions of A: 2x2 $\leftarrow$ $m_1=2$

Dimensions of B: 2x1 $\leftarrow$ $n_2=2$

Because m_1 is equal to n_2 the product CAN be done. The new matrix C will have dimensions 2x1

Review of Matrices
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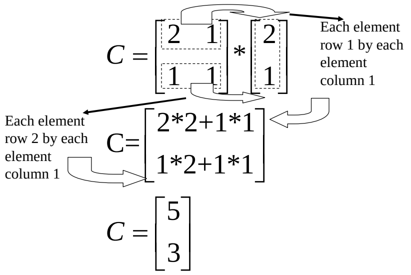
Solution Example Continues



$$C = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} * \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Each element row 1 by each element column 1

Each element row 2 by each element column 1

$$C = \begin{bmatrix} 2*2+1*1 \\ 1*2+1*1 \end{bmatrix}$$

$$C = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Exercise 5

- Given the matrices X and Y, calculate the matrix P = X Y

$$X = \begin{bmatrix} 3 & 1 \\ 8 & 6 \\ 0 & 4 \end{bmatrix} \quad \mathrm{Y} = \begin{bmatrix} 5 & 9 \\ 7 & 2 \end{bmatrix}$$

Review of Matrices
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Matrix Operations

- Operations of product of matrices
  - The product of two matrices is not commutative

$$AB \neq BA$$

  - The product of matrices is associative

$$(AB)C = A(BC)$$

  - The product of matrices is distributive

$$A(B+C) = (AB) + (AC)$$

Exercise 6

- Given the matrices A, B, and C, and the scalar g=2. Perform the following computation:

  D = g*A(B+C)

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

Review of Matrices
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Matrix Operations

- Transpose
  - The transpose “A^T” of a matrix “A” transforms rows into columns
  - Given a matrix “A”, the transpose matrix “A^T” is the matrix in which a_{ij} of the transpose is a_{ji} of the original matrix

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Review of Matrices
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Solution Example Continues

$$C = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

With the transpose property the columns of the matrix become the rows of the transpose

$$C^{'} = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

Exercise 7

- Given matrix A, calculate the transpose of A

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

Summary

- Can you define a matrix?
- Can you identify a particular element in a matrix?
- What are square matrices?
- What are vectors?
- Can you
  - Add matrices
  - Subtract matrices
  - Multiply matrices (remember product properties)
  - Calculate transpose of matrices